

# Learning for Single-Shot Confidence Calibration in Deep Neural Networks through Stochastic Inferences



Seonguk Seo\*<sup>1</sup>



Paul Hongsuck Seo\*<sup>1,2</sup>



Bohyung Han<sup>1</sup>



서울대학교

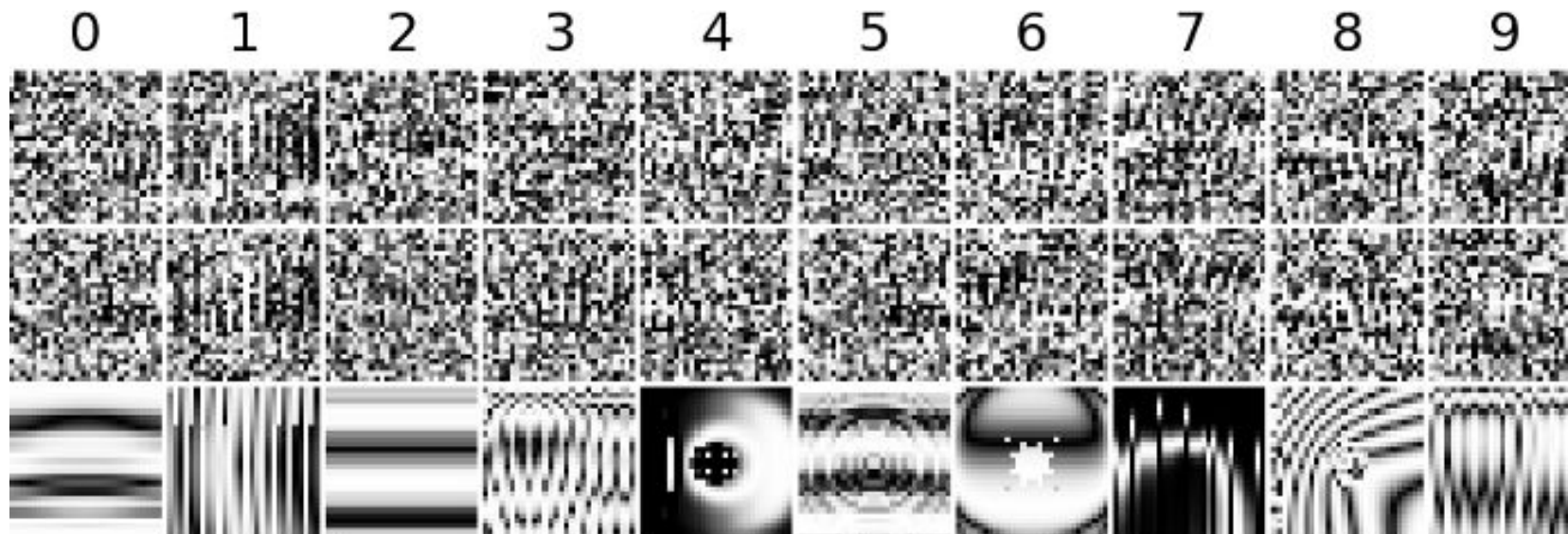
SEOUL NATIONAL UNIVERSITY

**POSTECH**

POHANG UNIVERSITY OF SCIENCE AND TECHNOLOGY

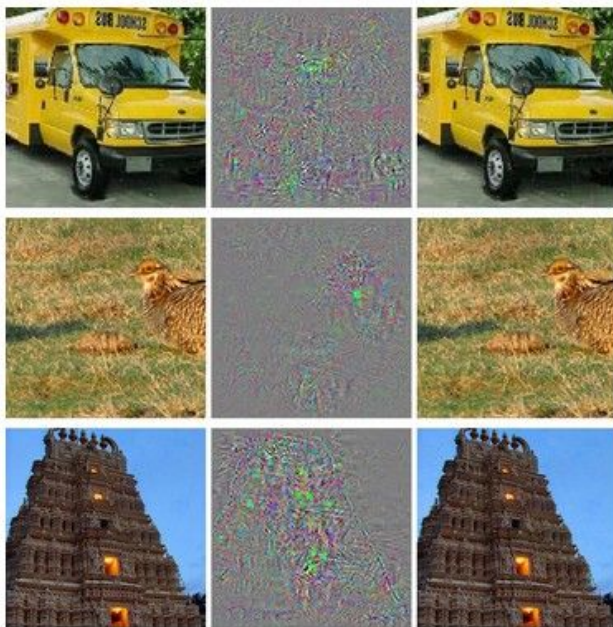
# Overconfidence Issues

- Overconfidence to unseen examples
  - 99.9+% sure for the following predictions



# Vulnerability

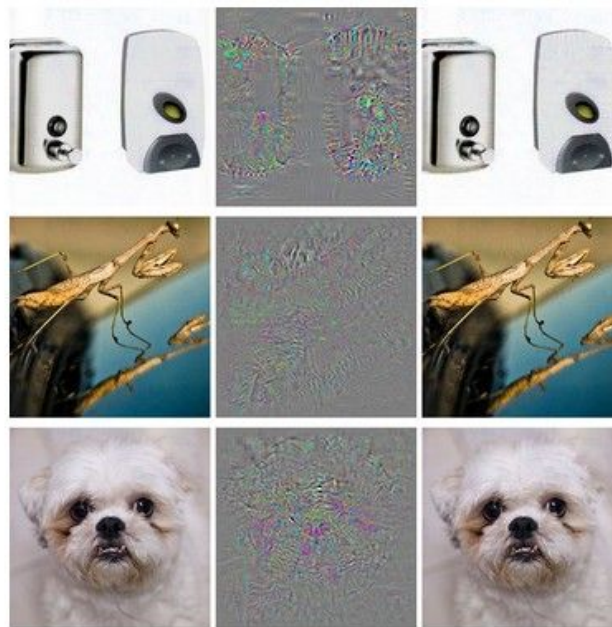
- Vulnerability to noise



Correct

Noise

Ostrich



Correct

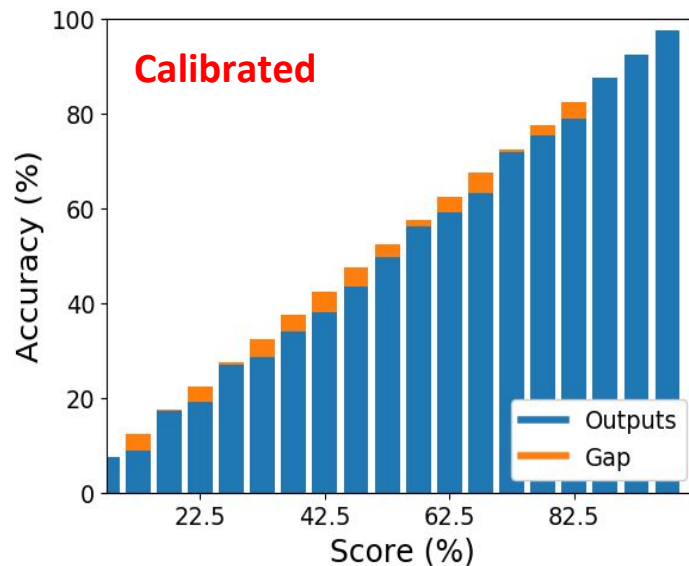
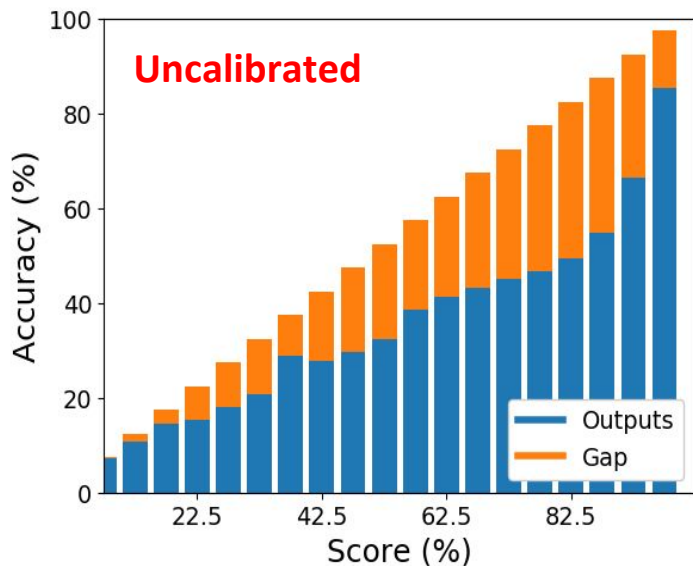
Noise

Ostrich



# Goals

- Confidence calibration
  - Reducing the discrepancy between confidence (score) and expected accuracy
  - Adopting idea of stochastic regularization



# Stochastic Regularization

- Regularization by noise: reducing overfitting problem by adding noise (randomness) to data or models
  - Noise injection to training data
  - Dropout<sup>[Srivastava14]</sup>
  - DropConnect<sup>[Wan13]</sup>
  - Learning with stochastic depth<sup>[Huang16]</sup>

[Srivastava14] N. Srivastava, G. E. Hinton, A. Krizhevsky, I. Sutskever, R. Salakhutdinov: **Dropout: a simple way to prevent neural networks from overfitting**. JMLR 2014

[Wan13] L. Wan, M. Zeiler, S. Zhang, Y. LeCun, R. Fergus. **Regularization of neural networks using dropconnect**. ICML 2013

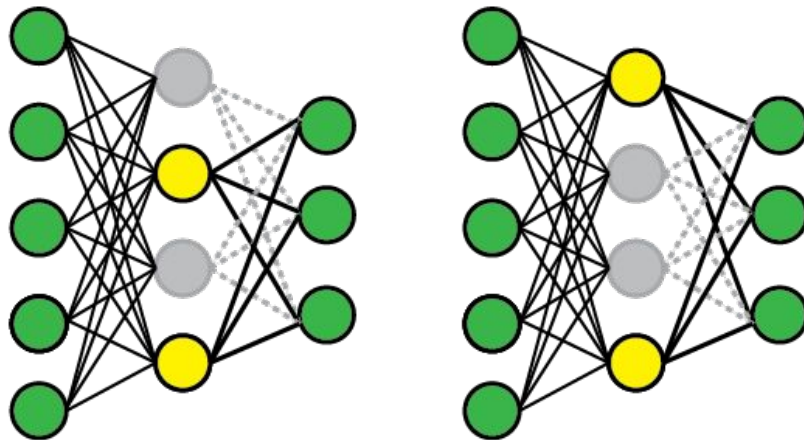
[Huang16] G. Huang, Y. Sun, Z. Liu, D. Sedra, K. Q. Weinberger: **Deep networks with stochastic depth**. ECCV 2016

# Stochastic Regularization

- Objective (in classification)
  - Perturbing parameters by element-wise multiplication during training

$$\hat{\mathcal{L}}_{\text{SR}}(\theta) = -\frac{1}{M} \sum_{i=1}^M \log p(y_i | x_i, \hat{\omega}_i) + \lambda \|\theta\|_2^2 \quad \text{where } \hat{\omega}_i = \theta \odot \epsilon_i$$

- Dropout

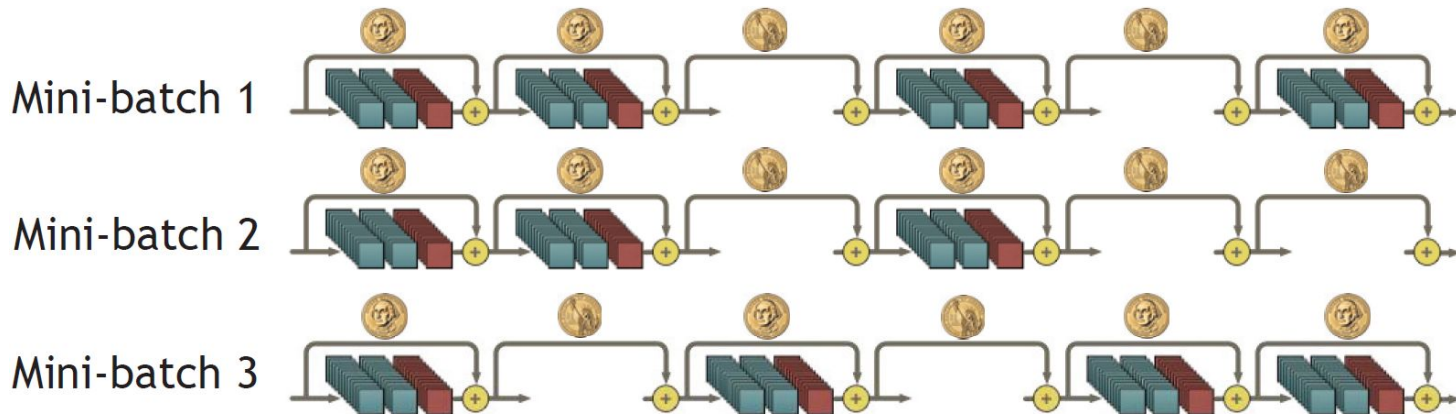


# Stochastic Regularization

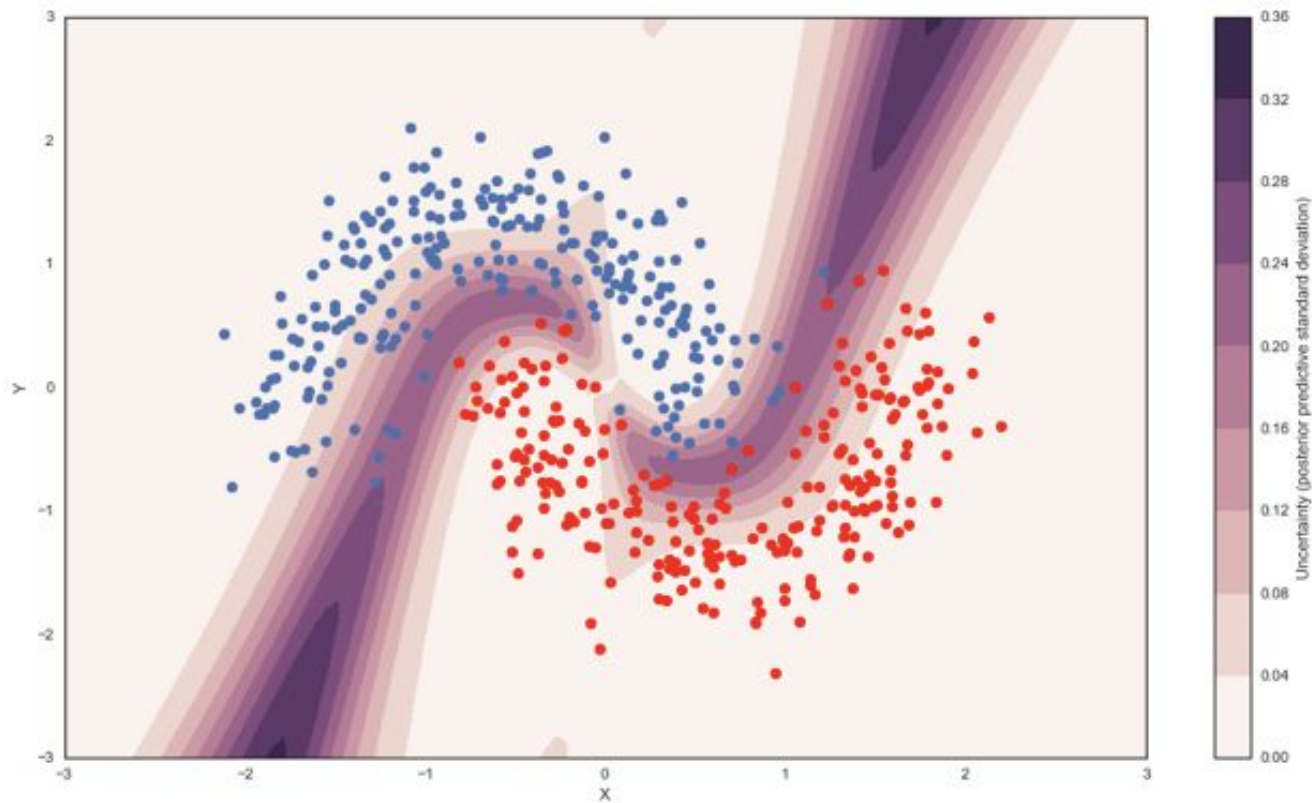
- Objective (in classification)
  - Perturbing parameters by element-wise multiplication during training

$$\hat{\mathcal{L}}_{\text{SR}}(\theta) = -\frac{1}{M} \sum_{i=1}^M \log p(y_i | x_i, \hat{\omega}_i) + \lambda \|\theta\|_2^2 \quad \text{where} \quad \hat{\omega}_i = \theta \odot \epsilon_i$$

- Stochastic depth



# Uncertainty in Deep Neural Networks





# Bayesian Uncertainty Estimation

- Integrating stochastic regularization techniques for inferences
  - Dropout, stochastic depth, etc.
  - Individual inferences produce different outputs.
- Uncertainty can be measured by multiple stochastic inferences.

# Bayesian Uncertainty Estimation

- Bayesian interpretation of stochastic regularization

- Learning objective: maximizing marginal likelihood by estimating posterior  $p(\omega|\mathcal{D})$

$$p(y|x, \mathcal{D}) = \int_{\omega} p(y|x, \omega)p(\omega|\mathcal{D})d\omega.$$

- Variational approximation (but intractable integration)

$$\mathcal{L}_{\text{VA}}(\theta) = - \sum_{i=1}^N \int_{\omega} q_{\theta}(\omega) \log p(y_i|x_i, \omega)d\omega + D_{\text{KL}}(q_{\theta}(\omega)||p(\omega))$$

- Variational approximation with Monte Carlo: by sampling  $\hat{\omega}_{i,j} \sim q_{\theta}(\omega)$

$$\hat{\mathcal{L}}_{\text{VA}}(\theta) = - \frac{N}{MS} \sum_{i=1}^M \sum_{j=1}^S \log p(y_i|x_i, \hat{\omega}_{i,j}) + D_{\text{KL}}(q_{\theta}(\omega)||p(\omega))$$

# Bayesian Uncertainty Estimation

- Bayesian interpretation of stochastic regularization
  - Variational approximation with Monte Carlo: by sampling  $\hat{\omega}_{i,j} \sim q_{\theta}(\omega)$

$$\hat{\mathcal{L}}_{\text{VA}}(\theta) = -\frac{N}{MS} \sum_{i=1}^M \sum_{j=1}^S \log p(y_i | x_i, \hat{\omega}_{i,j}) + D_{\text{KL}}(q_{\theta}(\omega) || p(\omega))$$

- Learning with stochastic regularization with weight decay: same objective with Gaussian assumption of true and approximated posteriors

$$\hat{\mathcal{L}}_{\text{SR}}(\theta) = -\frac{1}{M} \sum_{i=1}^M \log p(y_i | x_i, \hat{\omega}_i) + \lambda \|\theta\|_2^2;$$

- **The average prediction and its uncertainty can be computed directly from multiple stochastic inferences.**

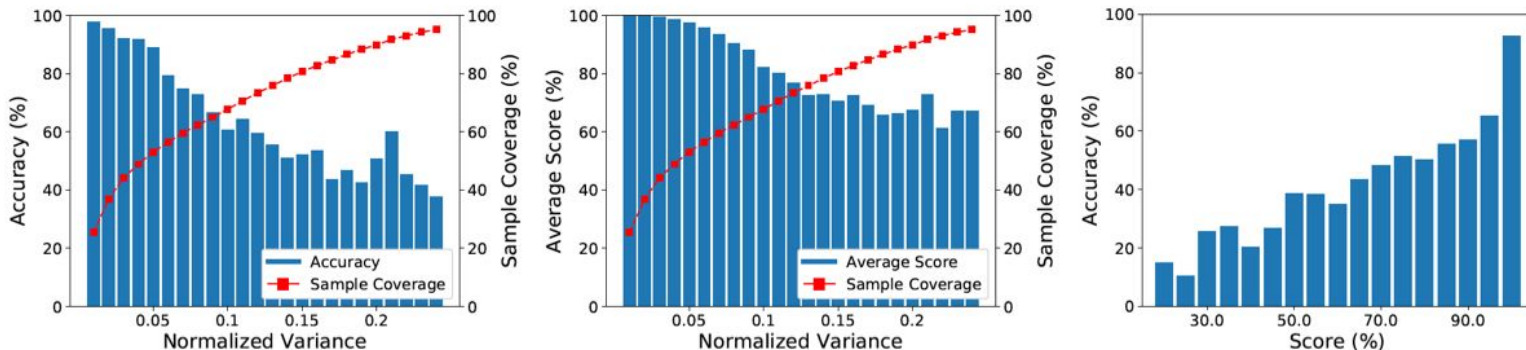
$$\mathbb{E}_{\hat{p}}[y = c] \approx \frac{1}{T} \sum_{i=1}^T \hat{p}(y = c | x, \hat{\omega}_i) \quad \text{and} \quad \text{Cov}_{\hat{p}}[\mathbf{y}] \approx \mathbb{E}_{\hat{p}}[\mathbf{y}\mathbf{y}^{\top}] - \mathbb{E}_{\hat{p}}[\mathbf{y}]\mathbb{E}_{\hat{p}}[\mathbf{y}]^{\top}$$

# Bayesian Uncertainty Estimation

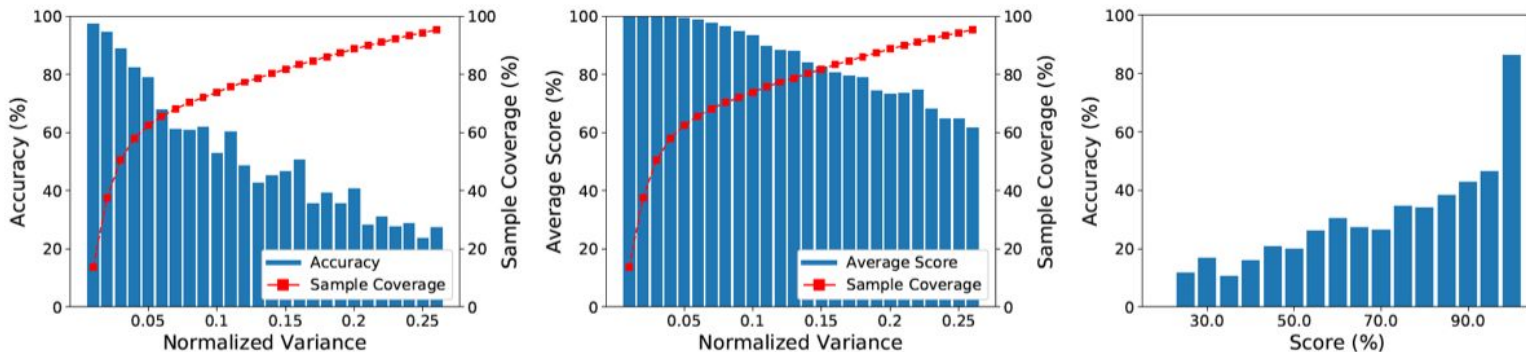
- Integrating stochastic regularization techniques for inferences
  - Dropout, stochastic depth, etc.
  - Individual inferences produce different outputs.
- Uncertainty can be measured by multiple stochastic inferences.

**The uncertainty of a prediction can be estimated using the variation of multiple stochastic inferences.**

# Empirical Observations



(a) Prediction uncertainty characteristics with stochastic depth in ResNet-34



(b) Prediction uncertainty characteristics with dropout in VGGNet with 16 layers

# Uncertainty through Stochastic Inferences

- Limitation of the simple uncertainty estimation method by multiple stochastic inferences
  - Requires multiple inferences for each example
- Solution
  - Designing a loss function to learn uncertainty
  - Exploiting multiple stochastic inferences results for training
  - Learning a model for the single-shot confidence calibration
- Desired score distribution
  - Confident examples have prediction scores close to one-hot vectors.
  - Uncertain examples produce relatively flat score distributions.

**We propose a loss function to make the confidence (the prediction score) proportional to the expected accuracy.**

# Confidence-Integrated Loss

- A naive loss function for accuracy-score calibration
  - A linear combination of two loss terms with respect to ground-truth and uniform distribution
  - Blindly augmenting a loss term with a uniform distribution

$$\begin{aligned}\mathcal{L}(\theta) &= \mathcal{L}_{\text{GT}}(\theta) + \beta \mathcal{L}_{\text{U}}(\theta) \\ &= \sum_{i=1}^N H(p_{\text{GT}}(y_i|x_i), p(y|x_i, \theta)) + \beta H(\mathcal{U}(y), p(y|x_i, \theta)) \\ &= \sum_{i=1}^N \boxed{-\log p(y_i|x_i, \theta)} + \beta \boxed{D_{\text{KL}}(\mathcal{U}(y) || p(y|x_i, \theta))} + \xi.\end{aligned}$$

**Accuracy term**                      **Confidence term**

# Confidence-Integrated Loss

- The same loss functions are discussed for different purposes
  - [Pereyra17]: for accuracy improved via regularization
  - [Lee18]: for identifying out-of-distribution examples
  - No attempt to estimate the confidence of predictions

$$\begin{aligned}\mathcal{L}(\theta) &= \mathcal{L}_{\text{GT}}(\theta) + \beta \mathcal{L}_{\text{U}}(\theta) \\ &= \sum_{i=1}^N H(p_{\text{GT}}(y_i|x_i), p(y|x_i, \theta)) + \beta H(\mathcal{U}(y), p(y|x_i, \theta)) \\ &= \sum_{i=1}^N -\log p(y_i|x_i, \theta) + \beta D_{\text{KL}}(\mathcal{U}(y) || p(y|x_i, \theta)) + \xi.\end{aligned}$$

[Pereyra17] G. Pereyra, G. Tucker, J. Chorowski, Ł. Kaiser, G. Hinton. **Regularizing neural networks by penalizing confident output distributions.** arXiv 2017

[Lee18] K. Lee, H. Lee, K. Lee, J. Shin. **Training confidence-calibrated classifiers for detecting out-of-distribution samples.** ICLR 2018



# Confidence-Integrated Loss

- A simple loss function for accuracy-score calibration
  - All samples have the same weight of the confidence loss term regardless of example-specific characteristics.
  - Interpretation of this loss function is very hard.
  - Needs for a global hyper-parameter  $\beta$

$$\begin{aligned}\mathcal{L}(\theta) &= \mathcal{L}_{\text{GT}}(\theta) + \beta \mathcal{L}_{\text{U}}(\theta) \\ &= \sum_{i=1}^N H(p_{\text{GT}}(y_i|x_i), p(y|x_i, \theta)) + \beta H(\mathcal{U}(y), p(y|x_i, \theta)) \\ &= \sum_{i=1}^N -\log p(y_i|x_i, \theta) + \beta D_{\text{KL}}(\mathcal{U}(y) || p(y|x_i, \theta)) + \xi.\end{aligned}$$

# Variance-Weighted Confidence-Integrated Loss

- A more sophisticated loss function for accuracy-score calibration
  - An interpolation of two cross-entropy terms
  - The two terms are weighted by the variance of stochastic inferences
  - Generalization of the confidence-integrated loss function

$$\begin{aligned}\mathcal{L}(\theta) &= \sum_{i=1}^N (1 - \alpha_i) \mathcal{L}_{\text{GT}}^{(i)}(\theta) + \alpha_i \mathcal{L}_{\text{U}}^{(i)}(\theta) \\ &= \frac{1}{T} \sum_{i=1}^N \sum_{j=1}^T -(1 - \alpha_i) \log p(y_i | x_i, \hat{w}_{i,j}) + \alpha_i D_{\text{KL}}(\mathcal{U}(y) || p(y | x_i, \hat{w}_{i,j})) + \xi_i\end{aligned}$$

$\alpha_i$ : normalized variance

# Variance-Weighted Confidence-Integrated Loss

- A more sophisticated loss function for accuracy-score calibration
  - Motivated by Bayesian interpretation of stochastic regularization and our empirical observation
  - No hyper-parameter to balance two terms

$$\begin{aligned}\mathcal{L}(\theta) &= \sum_{i=1}^N (1 - \alpha_i) \mathcal{L}_{\text{GT}}^{(i)}(\theta) + \alpha_i \mathcal{L}_{\text{U}}^{(i)}(\theta) \\ &= \frac{1}{T} \sum_{i=1}^N \sum_{j=1}^T -(1 - \alpha_i) \log p(y_i | x_i, \hat{w}_{i,j}) + \alpha_i D_{\text{KL}}(\mathcal{U}(y) || p(y | x_i, \hat{w}_{i,j})) + \xi_i\end{aligned}$$

$\alpha_i$ : normalized variance

# Experiments

- Datasets
  - CIFAR-100
  - Tiny ImageNet
- Architectures
  - ResNet
  - VGG
  - WideResNet
  - DenseNet

# Experiments

- Evaluation metrics

- Classification accuracy
- Calibration scores

- Expected Calibration Error (ECE): 
$$\text{ECE} = \sum_{m=1}^M \frac{B_m}{N'} |\text{acc}(B_m) - \text{conf}(B_m)|$$

- Maximum Calibration Error (MCE): 
$$\text{MCE} = \max_{m \in \{1, \dots, M\}} |\text{acc}(B_m) - \text{conf}(B_m)|$$

- Negative Log Likelihood (NLL): 
$$\text{NLL} = - \sum_{i=1}^{N'} \log p(y_i | x_i, \theta)$$

- Brier Score: 
$$\text{Brier} = - \sum_{i=1}^{N'} \sum_{j=1}^C (p(y_i = j | x_i, \theta) - \delta(y_i - j))^2$$

# Results

- On Tiny ImageNet

Dataset	Architecture	Method	Accuracy[%]	ECE	MCE	NLL	Brier Score
Tiny ImageNet	ResNet-34	Baseline	50.82	0.067	0.147	2.050	0.628
		CI	50.09 ± 1.08	0.134 ± 0.079	0.257 ± 0.098	2.270 ± 0.212	0.665 ± 0.037
		VWCI	<b>52.80</b>	<b>0.027</b>	<b>0.076</b>	<b>1.949</b>	<b>0.605</b>
		CI[Oracle]	51.45	0.035	0.171	2.030	0.620
		Baseline	46.58	0.346	0.595	4.220	0.844
	VGG-16	CI	46.82 ± 0.81	0.226 ± 0.095	0.435 ± 0.107	3.224 ± 0.468	0.761 ± 0.054
		VWCI	<b>48.03</b>	<b>0.053</b>	<b>0.142</b>	<b>2.373</b>	<b>0.659</b>
		CI[Oracle]	47.39	0.122	0.320	2.812	0.701
		Baseline	55.92	0.132	0.237	1.974	0.593
		WideResNet-16-8	CI	55.80 ± 0.44	0.115 ± 0.040	0.288 ± 0.100	1.980 ± 0.114
	VWCI		<b>56.66</b>	<b>0.046</b>	<b>0.136</b>	<b>1.866</b>	<b>0.569</b>
	CI[Oracle]		56.38	0.050	0.208	1.851	0.572
	Baseline		42.50	<b>0.020</b>	0.154	2.423	0.716
	DenseNet-40-12		CI	40.18 ± 1.68	0.059 ± 0.061	0.152 ± 0.082	2.606 ± 0.208
		VWCI	<b>43.25</b>	0.025	<b>0.089</b>	<b>2.410</b>	<b>0.712</b>
		CI[Oracle]	41.21	0.025	0.094	2.489	0.726

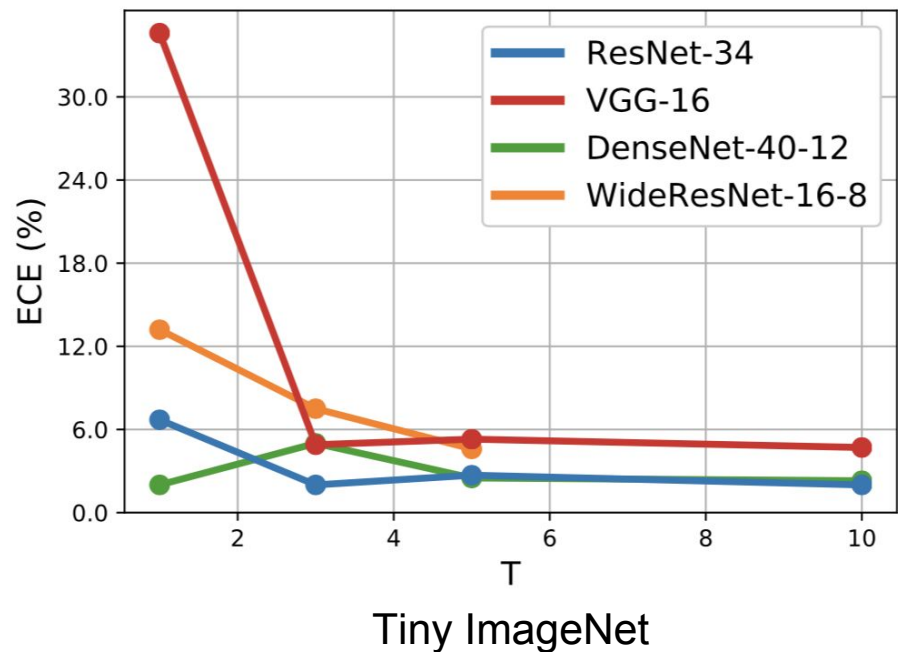
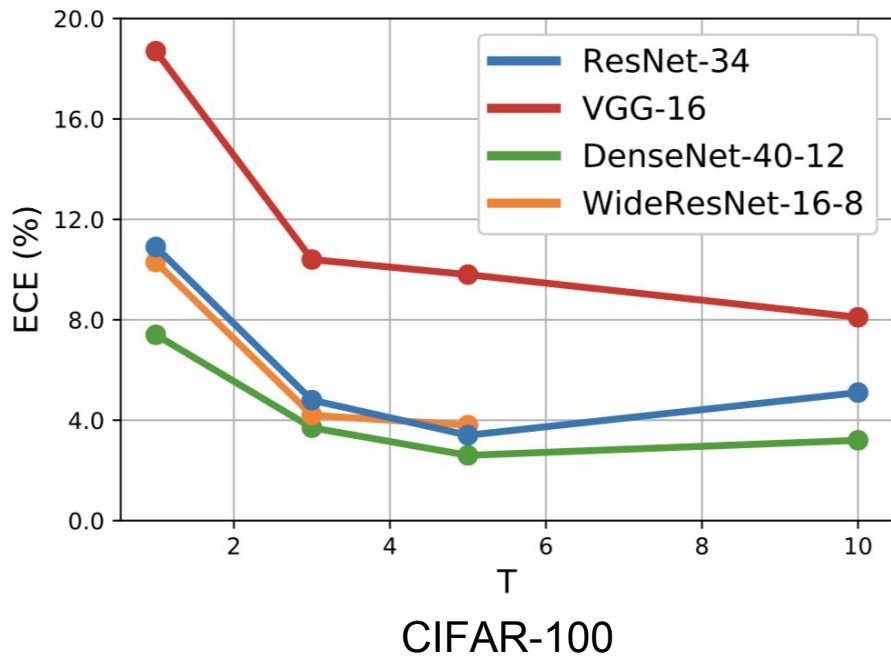
# Results

- On Tiny ImageNet

Dataset	Architecture	Method	Accuracy[%]	ECE	MCE	NLL	Brier Score
CIFAR-100	ResNet-34	Baseline	77.19	0.109	0.304	1.020	0.345
		CI	$77.56 \pm 0.60$	$0.134 \pm 0.131$	$0.251 \pm 0.128$	$1.064 \pm 0.217$	$0.360 \pm 0.057$
		VWCI	<b>78.64</b>	<b>0.034</b>	<b>0.089</b>	<b>0.908</b>	<b>0.310</b>
		CI[Oracle]	78.54	0.029	0.087	0.921	0.321
		Baseline	73.78	0.187	0.486	1.667	0.437
	VGG-16	CI	$73.75 \pm 0.35$	$0.183 \pm 0.079$	$0.489 \pm 0.214$	$1.526 \pm 0.175$	$0.436 \pm 0.034$
		VWCI	<b>73.87</b>	<b>0.098</b>	<b>0.309</b>	<b>1.277</b>	<b>0.391</b>
		CI[Oracle]	73.78	0.083	0.285	1.289	0.396
		Baseline	77.52	0.103	0.278	0.984	0.336
		WideResNet-16-8	CI	$77.35 \pm 0.21$	$0.133 \pm 0.091$	$0.297 \pm 0.108$	$1.062 \pm 0.180$
	VWCI		<b>77.74</b>	<b>0.038</b>	<b>0.101</b>	<b>0.891</b>	<b>0.314</b>
	CI[Oracle]		77.53	0.074	0.211	0.931	0.327
	Baseline		65.91	0.074	0.134	1.238	0.463
	DenseNet-40-12		CI	$64.72 \pm 1.46$	$0.070 \pm 0.040$	$0.138 \pm 0.055$	$1.312 \pm 0.125$
		VWCI	<b>67.45</b>	<b>0.026</b>	<b>0.094</b>	<b>1.161</b>	<b>0.439</b>
		CI[Oracle]	66.20	0.019	0.053	1.206	0.456

# Ablation Study

- Calibration performance w.r.t. the number of stochastic inferences during training





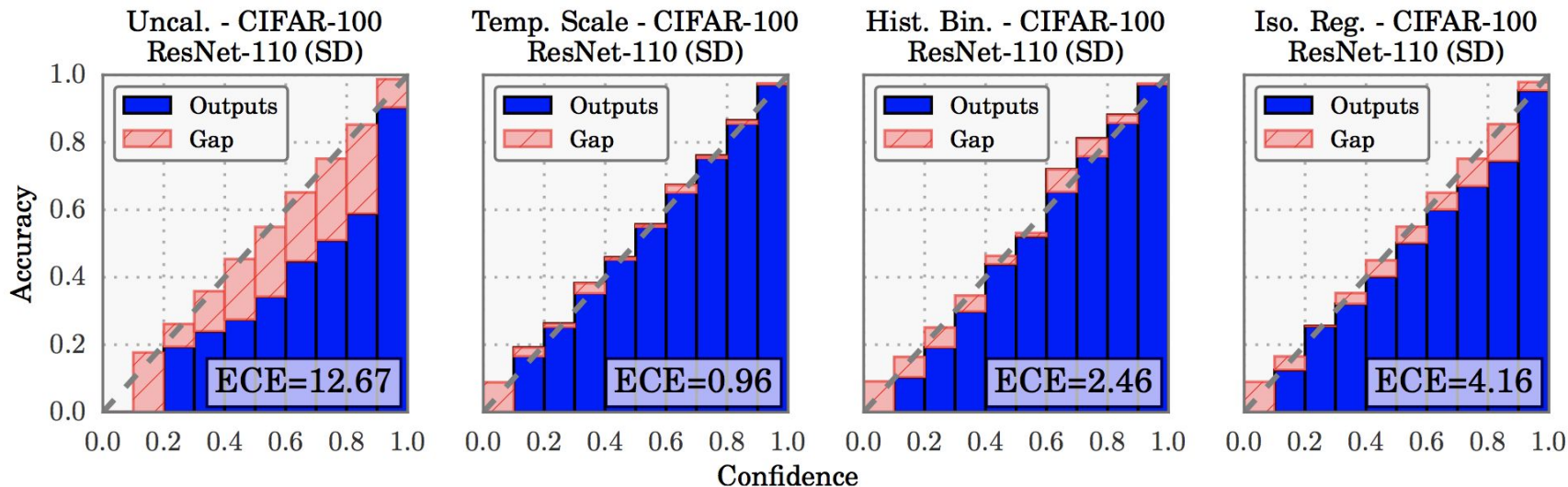
# Ablation Study

- Performance of the models fine-tuned with the VWCI losses
  - From the uncalibrated pretrained networks
  - On CIFAR-100
  - About 25% of the additional iterations are sufficient for good calibration.

Architecture	Method	Acc. [%]	ECE	MCE	NLL	Brier
VGG-16	Baseline	73.78	0.187	0.486	1.667	0.437
	VWCI	73.87	0.098	0.309	1.277	0.391
	Baseline → VWCI	<b>74.17</b>	<b>0.074</b>	<b>0.243</b>	<b>1.227</b>	<b>0.385</b>
ResNet-34	Baseline	77.19	0.109	0.304	1.020	0.345
	VWCI	<b>78.64</b>	0.034	0.089	<b>0.908</b>	<b>0.310</b>
	Baseline → VWCI	77.87	<b>0.026</b>	<b>0.069</b>	1.013	0.346

# Temperature Scaling

- A simple confidence calibration technique
  - Optimizes temperature of softmax function
  - Simple to implement and train
  - Does not change prediction results



# Results

- Comparison with temperature scaling<sup>[Guo17]</sup>
  - Case 1: using the entire training set for both training and calibration
  - Case 2: using 90% of training set for training and the rest for calibration
  - It may suffers from binning artifacts

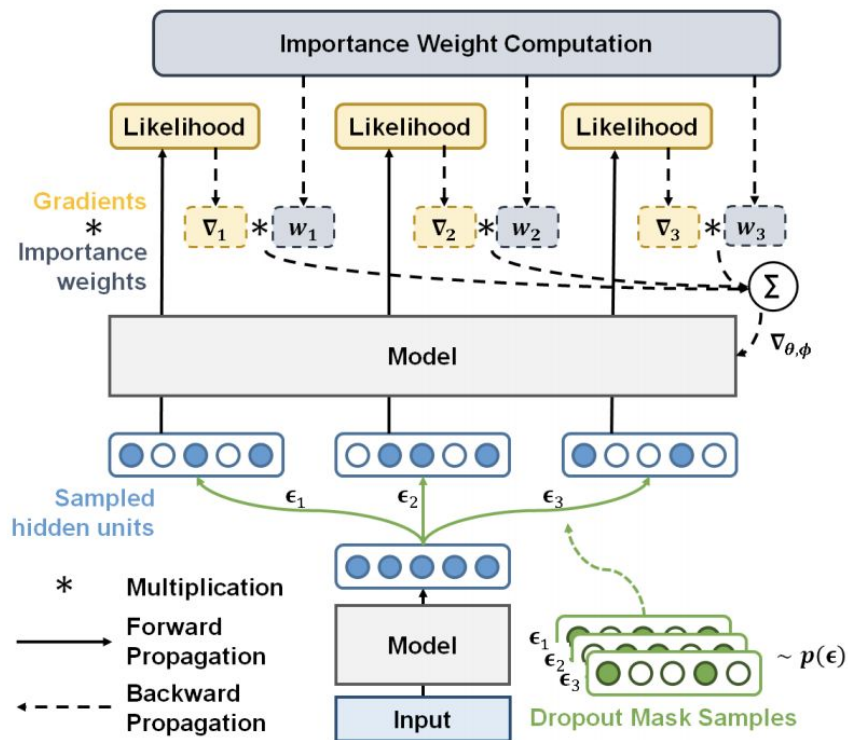
Dataset	Architecture	Method	Accuracy[%]	ECE	MCE	NLL	Brier Score
Tiny ImageNet	ResNet-34	TS (case 1)	50.82	0.162	0.272	2.241	0.660
		TS (case 2)	47.20	<b>0.021</b>	0.080	2.159	0.661
		VWCI	<b>52.80</b>	0.027	<b>0.076</b>	<b>1.949</b>	<b>0.605</b>
	VGG-16	TS (case 1)	46.58	0.358	0.604	4.425	0.855
		TS (case 2)	46.53	<b>0.028</b>	<b>0.067</b>	<b>2.361</b>	0.671
		VWCI	<b>48.03</b>	0.053	0.142	2.373	<b>0.659</b>
	WideResNet-16-8	TS (case 1)	55.92	0.200	0.335	2.259	0.627
		TS (case 2)	53.95	<b>0.027</b>	0.224	1.925	0.595
		VWCI	<b>56.66</b>	0.046	<b>0.136</b>	<b>1.866</b>	<b>0.569</b>
	DenseNet-40-12	TS (case 1)	42.50	0.037	0.456	2.436	0.717
		TS (case 2)	41.63	<b>0.024</b>	0.109	2.483	0.728
		VWCI	<b>43.25</b>	0.025	<b>0.089</b>	<b>2.410</b>	<b>0.712</b>

# Summary on Confidence Calibration

- A Bayesian interpretation of generic stochastic regularization techniques with multiplicative noise
- A generic framework to calibrate accuracy and confidence (score) of a prediction
  - Through stochastic inferences in deep neural networks
  - Introducing Variance-Weighted Confidence-Integrated (VWCI) loss
  - Capable of estimating prediction uncertainty using a single prediction
  - Supported by empirical observations
- Promising and consistent performance on multiple datasets and stochastic inference techniques

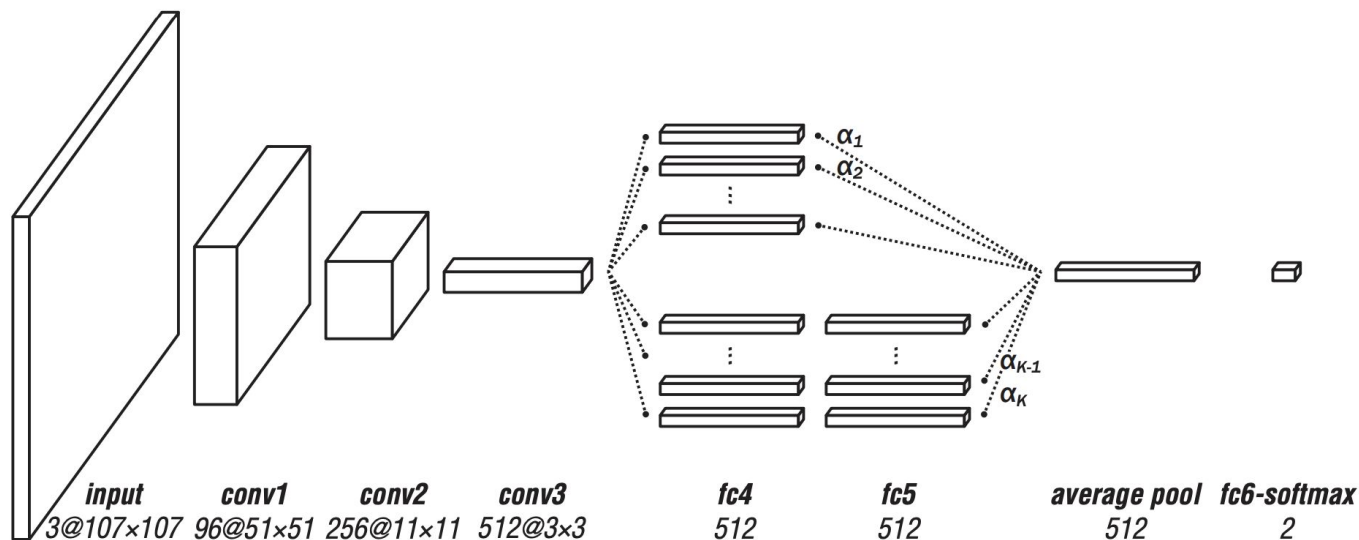
# Other Works Related to Stochastic Learning

- Regularization by noise
  - Sampling multiple dropout masks
  - Learning with importance weighted stochastic gradient
- Interpretation and benefit
  - Improving the lower-bound of marginal likelihood by increasing the number of samples
  - Better accuracy in several domains



# Other Works Related to Stochastic Learning

- Stochastic online few-shot ensemble learning
  - Preventing correlation of representations obtained from multiple branches
  - Randomly selecting branches for updates



# Other Research (in ML Perspective)

- Weakly supervised learning<sup>[NIPS2015, CVPR2016, AAAI2017, CVPR2017a, CVPR2018]</sup>
- Multi-modal learning<sup>[CVPR2016, AAAI2017, ICCV2017, NIPS2017]</sup>
- Metric learning<sup>[CVPR2017b]</sup>
- Multiple choice learning<sup>[NeurIPS2018]</sup>
- Zero-shot transfer learning<sup>[arXiv2018]</sup>
- Combinatorial learning
- Meta-learning
- Continual learning
- AutoML