Knowledge Tracing Machines: Factorization Machines for Knowledge Tracing

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https://arxiv.org/abs/1811.03388

Practical intro

When exercises are too easy/difficult,
students get bored/discouraged.

To personalize assessment,
$\Rightarrow$ need a model of how people respond to exercises.

Example

To personalize this presentation,
$\Rightarrow$ need a model of how people respond to my slides.

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p(understanding)
Practical: 0.9
Theoretical: 0.6

Theoretical intro

Let us assume $\boldsymbol{x}$ is sparse.

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Linear regression $y = \langle \boldsymbol{w}, \boldsymbol{x} \rangle$

Logistic regression $y = \sigma(\langle \boldsymbol{w}, \boldsymbol{x} \rangle)$ where σ is sigmoid.

Neural network $x^{(L+1)} = \sigma(\langle \boldsymbol{w}, \boldsymbol{x}^{(L)} \rangle)$ where σ is ReLU.

What if $\sigma : x \mapsto x^2$ for example?

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What if $\sigma : x \mapsto x^2$ for example?

Polynomial kernel $y = \sigma(1 + \langle \boldsymbol{w}, \boldsymbol{x} \rangle)$ where σ is a monomial.

Factorization machine $y = \langle \boldsymbol{w}, \boldsymbol{x} \rangle + ||V\boldsymbol{x}||^2$

Mathieu Blondel, Masakazu Ishihata, Akinori Fujino, and Naonori Ueda (2016). "Polynomial networks and factorization machines: new insights and efficient training algorithms". In: *Proceedings of the 33rd International Conference on International Conference on Machine Learning-Volume 48*. JMLR. org, pp. 850–858

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Students try exercises

Math Learning

Items	$5 - 5 = ?$
New student	○

Students try exercises

Math Learning

Items	$5 - 5 = ?$	$17 - 3 = ?$
New student	○	○

Students try exercises

Math Learning

Items	$5 - 5 = ?$	$17 - 3 = ?$	$13 - 7 = ?$
New student	○	○	×

Students try exercises

Math Learning

Items	$5 - 5 = ?$	$17 - 3 = ?$	$13 - 7 = ?$
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Language Learning

	PRON	VERB	PRON	NOUN	CONJ	PRON	VERB	PRON	NOUN
correct:	She	is	my	mother	and	he	is	my	father
student:	she	is		mader	and	he	is		fhader
label:	O	O	✖	✖	O	O	O	✖	✖

Students try exercises

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New student	○	○	×

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Challenges

- Users can attempt a same item multiple times
- Users learn over time
- People can make mistakes that do not reflect their knowledge

Predicting student performance: knowledge tracing

Data

A population of users answering items

- Events: "User *i* answered item *j* correctly/incorrectly"

Side information

- If we know the skills required to solve each item *e.g.*, $+$, $\times$
- Class ID, school ID, etc.

Goal: classification problem

Predict the performance of new users on existing items\ Metric: AUC

Method

Learn parameters of questions from historical data *e.g.*, *difficulty*
Measure parameters of new students *e.g.*, *expertise*

Existing work

Model	Basically	Original AUC
Bayesian Knowledge Tracing (Corbett and Anderson 1994)	Hidden Markov Model	0.67

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$$\underbrace{\text{PFA}}_{\text{LogReg}} \leq \underbrace{\text{DKT}}_{\text{LSTM}} \leq \underbrace{\text{IRT}}_{\text{LogReg}} \leq \underbrace{\text{KTM}}_{\text{FM}}$$

Limitations and contributions

- Several models for knowledge tracing were developed independently
- In our paper, we prove that our approach is more generic

Our contributions

- Knowledge Tracing Machines unify most existing models
  - Encoding student data to sparse features
  - Then running logistic regression or factorization machines
- Better models found
  - It is better to estimate a bias per item, not only per skill
  - Side information improves performance more than higher dim.

Our small dataset

- User 1 answered Item 1 correct
- User 1 answered Item 2 incorrect
- User 2 answered Item 1 incorrect
- User 2 answered Item 1 correct
- User 2 answered Item 2 ???

user	item	correct
1	1	1
1	2	0
2	1	0
2	1	1
2	2	???

`dummy.csv`

Our approach

- Encode data to sparse features

user	item	correct
2	2	1
2	2	0
2	2	0
2	3	0
2	3	1
1	2	???
1	1	???

data.csv

encode →

KTM

PFA

IRT

Users		Items			Skills			Wins			Fails		
1	2	Q_1	Q_2	Q_3	KC_1	KC_2	KC_3	KC_1	KC_2	KC_3	KC_1	KC_2	KC_3
0	1	0	1	0	1	1	0	0	0	0	0	0	0
0	1	0	1	0	1	1	0	1	1	0	0	0	0
0	1	0	1	0	1	1	0	1	1	0	1	1	0
0	1	0	0	1	0	1	1	0	2	0	0	1	0
0	1	0	0	1	0	1	1	0	2	0	0	2	1
1	0	0	1	0	1	1	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0	0	0	0	0

sparse matrix X

- Run logistic regression or factorization machines
  $\Rightarrow$ recover existing models or better models

Model 1: Item Response Theory

Learn abilities θ_i for each user i
Learn easiness e_j for each item j such that:

$$Pr(\text{User } i \text{ Item } j \text{ OK}) = \sigma(\theta_i + e_j) \quad \sigma : x \mapsto 1/(1 + \exp(-x))$$
$$\operatorname{logit} Pr(\text{User } i \text{ Item } j \text{ OK}) = \theta_i + e_j$$

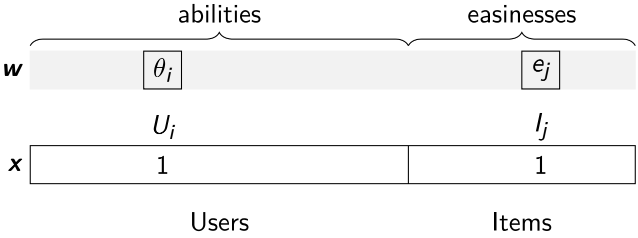
Really popular model, used for the PISA assessment

Logistic regression

Learn $\boldsymbol{w}$ such that $\operatorname{logit} Pr(\boldsymbol{x}) = \langle \boldsymbol{w}, \boldsymbol{x} \rangle + b$

Graphically: IRT as logistic regression

Encoding "User i answered Item j" with sparse features:



$$\langle \boldsymbol{w}, \boldsymbol{x} \rangle = \theta_i + e_j = \mathrm{logit}\, Pr(\text{User } i \text{ Item } j \text{ OK})$$

Encoding into sparse features

Users			Items		
U_0	U_1	U_2	I_0	I_1	I_2
0	1	0	0	1	0
0	1	0	0	0	1
0	0	1	0	1	0
0	0	1	0	1	0
0	0	1	0	0	1

Then logistic regression can be run on the sparse features.

Oh, there's a problem

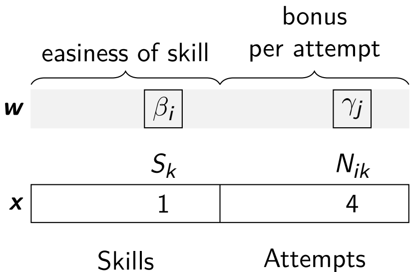
	Users			Items				
	U_0	U_1	U_2	I_0	I_1	I_2	$\boldsymbol{y}_{\text{pred}}$	$\boldsymbol{y}$
User 1 Item 1 OK	0	1	0	0	1	0	0.575135	1
User 1 Item 2 NOK	0	1	0	0	0	1	0.395036	0
User 2 Item 1 NOK	0	0	1	0	1	0	0.545417	0
User 2 Item 1 OK	0	0	1	0	1	0	0.545417	1
User 2 Item 2 NOK	0	0	1	0	0	1	0.366595	0

We predict the same thing when there are several attempts.

Count number of attempts: AFM

Keep a counter of attempts at skill level:

user	item	skill	correct	attempts (for the same skill)
1	1	1	1	0
1	2	2	0	0
2	1	1	0	0
2	1	1	1	1
2	2	2	0	0



Count successes and failures: PFA

Count separately successes W_{ik} and fails F_{ik} of student i over skill k.

user	item	skill	correct	wins	fails
1	1	1	1	0	0
1	2	2	0	0	0
2	1	1	0	0	0
2	1	1	1	0	1
2	2	2	0	0	0

	easiness of skill	bonus per success	bonus per failure
$\boldsymbol{w}$	β_i	γ_j	δ_j
	S_k	W_{ik}	F_{ik}
$\boldsymbol{x}$	1	1	1
	Skills	Wins	Fails

Model 2: Performance Factor Analysis

W_{ik}: how many successes of user i over skill k (F_{ik}: #failures)

Learn β_k, γ_k, δ_k for each skill k such that:

$$\text{logit } Pr(\text{User } i \text{ Item } j \text{ OK}) = \sum_{\text{Skill } k \text{ of Item } j} \beta_k + W_{ik}\gamma_k + F_{ik}\delta_k$$

Skills			Wins			Fails		
S_0	S_1	S_2	S_0	S_1	S_2	S_0	S_1	S_2
0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	1	0	0	0	0	0	1	0
0	0	1	0	0	0	0	0	0

Better!

	Skills			Wins			Fails				
	S_0	S_1	S_2	S_0	S_1	S_2	S_0	S_1	S_2	y_{pred}	y
User 1 Item 1 OK	0	1	0	0	0	0	0	0	0	0.544	1
User 1 Item 2 NOK	0	0	1	0	0	0	0	0	0	0.381	0
User 2 Item 1 NOK	0	1	0	0	0	0	0	0	0	0.544	0
User 2 Item 1 OK	0	1	0	0	0	0	0	1	0	0.633	1
User 2 Item 2 NOK	0	0	1	0	0	0	0	0	0	0.381	0

Test on a large dataset: Assistments 2009

346860 attempts of 4217 students over 26688 items on 123 skills.

model	dim	AUC	improvement
PFA: skills, wins, fails	0	0.685	+0.07
AFM: skills, attempts	0	0.616	

Model 3: a new model (but still logistic regression)

model	dim	AUC	improvement
KTM: items, skills, wins, fails	0	0.746	+0.06
IRT: users, items	0	0.691	
PFA: skills, wins, fails	0	0.685	+0.07
AFM: skills, attempts	0	0.616	

Here comes a new challenger

How to model pairwise interactions with side information?

Logistic Regression

Learn a 1-dim bias for each feature (each user, item, etc.)

Factorization Machines

Learn a 1-dim bias and a k-dim embedding for each feature

How to model pairwise interactions with side information?

If you know user i attempted item j on mobile (not desktop)
How to model it?

y: score of event "user i solves correctly item j"

IRT

$$y = \theta_i + e_j$$

Multidimensional IRT (similar to collaborative filtering)

$$y = \theta_i + e_j + \langle \boldsymbol{v}_{\text{user } \boldsymbol{i}}, \boldsymbol{v}_{\text{item } \boldsymbol{j}} \rangle$$

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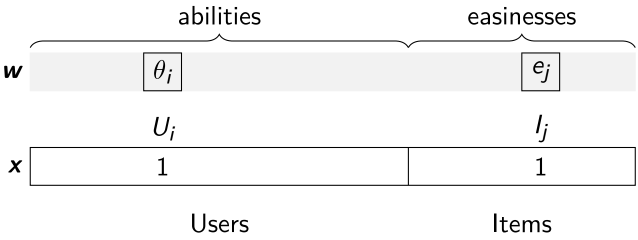
$$y = \theta_i + e_j + \langle \boldsymbol{v}_{\text{user } \boldsymbol{i}}, \boldsymbol{v}_{\text{item } \boldsymbol{j}} \rangle$$

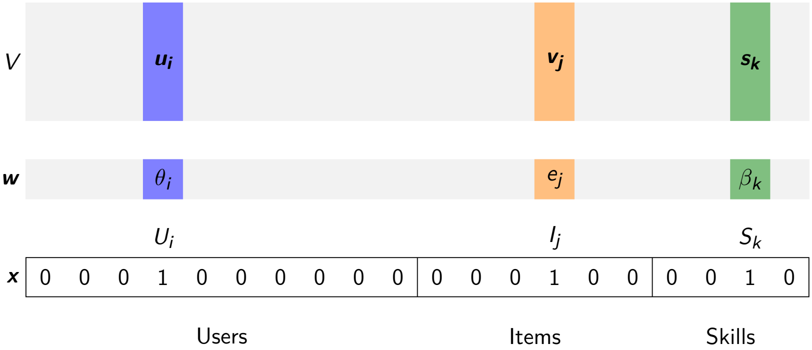
With side information

$$y = \theta_i + e_j + w_{\text{mobile}} + \langle \boldsymbol{v}_{\text{user } \boldsymbol{i}}, \boldsymbol{v}_{\text{item } \boldsymbol{j}} \rangle + \langle \boldsymbol{v}_{\text{user } \boldsymbol{i}}, \boldsymbol{v}_{\text{mobile}} \rangle + \langle \boldsymbol{v}_{\text{item } \boldsymbol{j}}, \boldsymbol{v}_{\text{mobile}} \rangle$$

Graphically: logistic regression

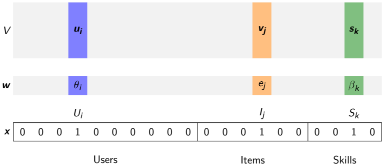


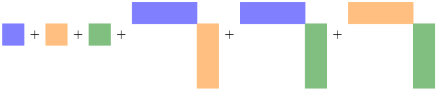
Graphically: factorization machines



Formally: factorization machines

Each user, item, skill k is modeled by bias w_k and embedding $\boldsymbol{v_k}$.





$$\operatorname{logit} p(\boldsymbol{x}) = \mu + \underbrace{\sum_{k=1}^{N} w_k x_k}_{\text{logistic regression}} + \underbrace{\sum_{1 \leq k < l \leq N} x_k x_l \langle \boldsymbol{v_k}, \boldsymbol{v_l} \rangle}_{\text{pairwise relationships}}$$

Steffen Rendle (2012). "Factorization Machines with libFM". In: *ACM Transactions on Intelligent Systems and Technology (TIST)* 3.3, 57:1–57:22. DOI: 10.1145/2168752.2168771

Training using MCMC

Priors: $w_k \sim \mathcal{N}(\mu_0, 1/\lambda_0)$ $\boldsymbol{v_k} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Lambda}^{-1})$
Hyperpriors: $\mu_0, \ldots, \mu_n \sim \mathcal{N}(0, 1), \lambda_0, \ldots, \lambda_n \sim \Gamma(1, 1) = U(0, 1)$

Algorithm 1 MCMC implementation of FMs

```
for each iteration do
    Sample hyperp. (λ_i, μ_i)_i from posterior using Gibbs sampling
    Sample weights w
    Sample vectors V
    Sample predictions y
end for
```

Implementation in C++ (libFM) with Python wrapper (pyWFM).

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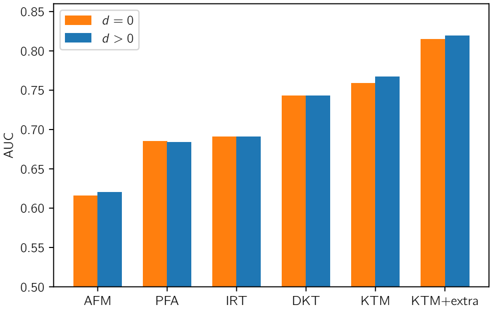
Datasets

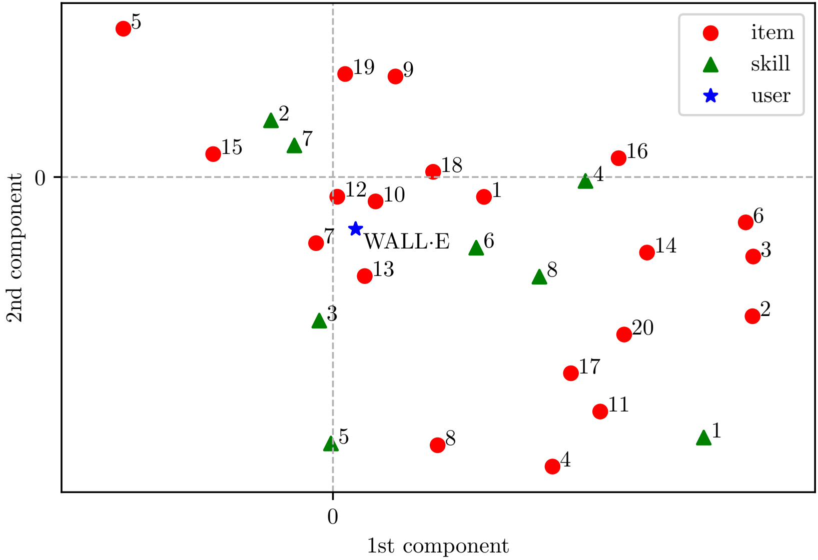
Name	Users	Items	Skills	Skills/i	Entries	Sparsity	Attempts/u
fraction	536	20	8	2.800	10720	0.000	1.000
timss	757	23	13	1.652	17411	0.000	1.000
ecpe	2922	28	3	1.321	81816	0.000	1.000
assistments	4217	26688	123	0.796	346860	0.997	1.014
berkeley	1730	234	29	1.000	562201	0.269	1.901
castor	58939	17	2	1.471	1001963	0.000	1.000

AUC results on the Assistments dataset



model	dim	AUC	improvement
KTM: items, skills, wins, fails, extra	5	0.819	
KTM: items, skills, wins, fails, extra	0	0.815	+0.05
KTM: items, skills, wins, fails	10	0.767	
KTM: items, skills, wins, fails	0	0.759	+0.02
DKT (Wilson et al., 2016)	100	0.743	+0.05
IRT: users, items	0	0.691	
PFA: skills, wins, fails	0	0.685	+0.07
AFM: skills, attempts	0	0.616	

Bonus: interpreting the learned embeddings

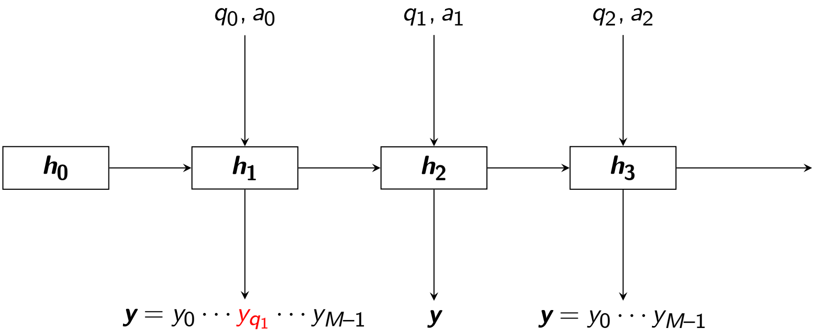


What 'bout recurrent neural networks?

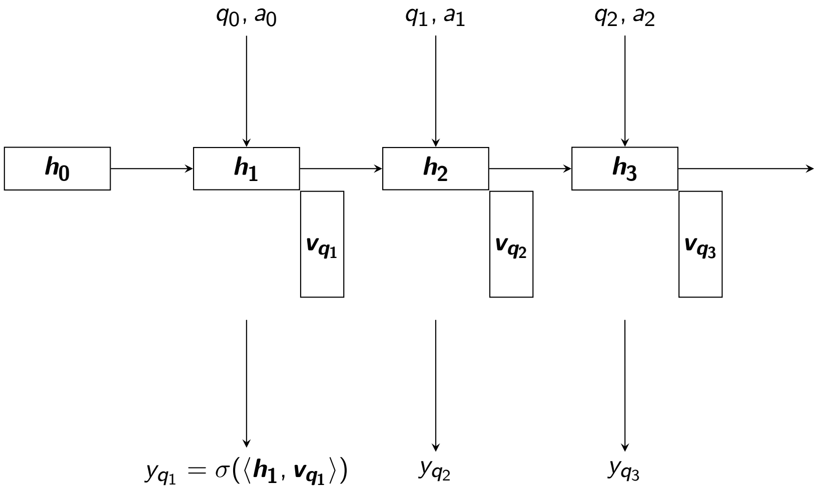
Deep Knowledge Tracing: model the problem as sequence prediction

- Each student on skill q_t has performance a_t
- How to predict outcomes $\boldsymbol{y}$ on every skill k?
- Spoiler: by measuring the evolution of a latent state $\boldsymbol{h_t}$

Graphically: deep knowledge tracing



Graphically: there is a MIRT in my DKT



Drawback of Deep Knowledge Tracing

DKT does not model individual differences.

Actually, Wilson even managed to beat DKT with (1-dim!) IRT.

By estimating on-the-fly the student's learning ability, we managed to get a better model.

AUC	BKT	IRT	PFA	DKT	DKT-DSC
Assistments 2009	0.67	0.75	0.70	0.73	0.91
Assistments 2012	0.61	0.74	0.67	0.72	0.87
Assistments 2014	0.64	0.67	0.69	0.72	0.87
Cognitive Tutor	0.61	0.81	0.76	0.79	0.81

Sein Minn, Yi Yu, Michel Desmarais, Feida Zhu, and Jill-Jênn Vie (2018). "Deep Knowledge Tracing and Dynamic Student Classification for Knowledge Tracing". In: *Proceedings of the 18th IEEE International Conference on Data Mining*, to appear. URL: https://arxiv.org/abs/1809.08713

Take home message

Knowledge tracing machines unify many existing EDM models

- Side information improves performance more than higher d
- We can visualize learning (and provide feedback to learners)

Already provides better results than vanilla deep neural networks

- Can be combined with FMs

Do you have any questions?

Read our article:

Knowledge Tracing Machines

https://arxiv.org/abs/1811.03388

Try our tutorial:

https://github.com/jilljenn/ktm

I'm interested in:

- predicting student performance
- recommender systems
- optimizing human learning using reinforcement learning

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Blondel, Mathieu, Masakazu Ishihata, Akinori Fujino, and Naonori Ueda (2016). "Polynomial networks and factorization machines: new insights and efficient training algorithms". In: *Proceedings of the 33rd International Conference on International Conference on Machine Learning-Volume 48*. JMLR. org, pp. 850–858.

Corbett, Albert T and John R Anderson (1994). "Knowledge tracing: Modeling the acquisition of procedural knowledge". In: *User modeling and user-adapted interaction* 4.4, pp. 253–278.

Minn, Sein, Yi Yu, Michel Desmarais, Feida Zhu, and Jill-Jênn Vie (2018). "Deep Knowledge Tracing and Dynamic Student Classification for Knowledge Tracing". In: *Proceedings of the 18th IEEE International Conference on Data Mining*, to appear. URL: https://arxiv.org/abs/1809.08713.

Piech, Chris, Jonathan Bassen, Jonathan Huang, Surya Ganguli, Mehran Sahami, Leonidas J Guibas, and Jascha Sohl-Dickstein (2015). "Deep knowledge tracing". In: *Advances in Neural Information Processing Systems (NIPS)*, pp. 505–513.

Rasch, Georg (1960). "Studies in mathematical psychology: I. Probabilistic models for some intelligence and attainment tests.". In:

Rendle, Steffen (2012). "Factorization Machines with libFM". In: *ACM Transactions on Intelligent Systems and Technology (TIST)* 3.3, 57:1–57:22. DOI: 10.1145/2168752.2168771.